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AVERAGED HEAT TRANSFER DURING PERIODIC FLUCTUATIONS OF THE HEAT TRANSFER INTENSITY ON THE SURFACE OF A PLATE, A CYLINDER, OR A SPHERE

Yu. B. Zudin

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Processes of heat transfer with periodically varying intensity on the surface of heated bodies of three typical geometries (plate, cylinder, sphere) are considered. The true heat transfer coefficient, which varies in time by the law of a periodic step function having two free parameters – amplitude and asymmetry, is specified on the heat transfer surface. Resultant relations are obtained for calculating the experimental heat transfer coefficient, which is the quantity measured in a traditional heat transfer experiment and used in applied calculations.

Processes of Heat Transfer with a Periodic Intensity. Real processes of heat transfer between a wall and a fluid are almost always accompanied by periodic fluctuations of the thermohydraulic parameters of the fluid (velocities, pressures, temperatures) relative to their mean values.

In [1] a general approximate method was developed to analyze processes of heat transfer having a periodic intensity. The method is based on replacement of the real complex picture of thermohydraulic fluctuations in the heat carrier by a simplified scheme with a prescribed "true" heat transfer coefficient that varies periodically along the heat transfer surface and in time:

$$\alpha = \langle \alpha \rangle (1 + \psi) . \tag{1}$$

Here $\langle \alpha \rangle$, ψ are the averaged and fluctuational components of the "true" heat transfer coefficient, respectively. By solving the nonstationary heat conduction equation for a wall with a boundary condition of the third kind we determine the temperature head ϑ and the heat flux density q in the wall and, in particular, those on the heat transfer surface ϑ_δ, q_δ. By definition the "true" heat transfer coefficient is equal to

$$\alpha = \frac{q_\delta}{\vartheta_\delta} = \frac{\langle q_\delta \rangle (1 + \tilde{q}_\delta')}{\langle \vartheta_\delta \rangle (1 + \tilde{\vartheta}_\delta')} , \tag{2}$$

where $\langle \vartheta_\delta \rangle$, $\langle q_\delta \rangle$ are the averaged, and $\tilde{\vartheta}_\delta$, $\tilde{q}_\delta$ are the reduced fluctuational, values of the quantities.

The value obtained as the quotient from division of the averaged heat flux density transmitted through the wall $\langle q_\delta \rangle$ by the averaged difference between the temperatures of the wall and the heat carrier $\langle \vartheta_\delta \rangle$ will be referred to as the "experimental" heat transfer coefficient [1]:

$$\alpha_m = \langle q_\delta \rangle / \langle \vartheta_\delta \rangle . \tag{3}$$

Note that precisely the quantity α_m is the objective of a traditional heat transfer experiment and is used in applied calculations. According to Eq. (2), the "true" averaged heat transfer coefficient is equal to

$$\langle \alpha \rangle = \langle \frac{q_\delta}{\vartheta_\delta} \rangle = \frac{\langle q_\delta \rangle}{\langle \vartheta_\delta \rangle} \langle \frac{1 + \tilde{q}_\delta'}{1 + \tilde{\vartheta}_\delta'} \rangle . \tag{4}$$

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Thus, the two different procedures for averaging Eq. (2) lead to two corresponding averaged values of the heat transfer coefficient: "true" $\langle \alpha \rangle$ and "experimental" α_m, which generally are unequal.

In [1] it is proved in a general form that the ratio of the indicated quantities $\varepsilon = \alpha_m/\langle \alpha \rangle$ varies within the limits

$$\langle \frac{1}{1+\psi} \rangle^{-1} \leq \varepsilon \leq 1 . \tag{5}$$

Thus, the "experimental" heat transfer coefficient is smaller than the "true" averaged one or is equal to it in the limit ($\alpha_m \leq \langle \alpha \rangle$).

As shown in [1] on specific examples, the "true" heat transfer coefficient in a variety of situations has time to "adapt" to changes in the parameters of the heat carrier and acquires the corresponding quasistationary values. This means that we may prescribe the quantity α as a function of the hydrodynamics of the flow, i.e., irrespective of the thermal effect of the wall. Then, by changing the wall parameters (the thermophysical properties, thickness, geometry), for α = idem we obtain a set of values of α_m that quantitatively and qualitatively expresses the thermal effect of the wall on the averaged heat transfer. The result of the analysis in [1] is the relative quantity $\varepsilon = \alpha_m/\langle \alpha \rangle$, which is a correcting factor for the averaged heat transfer coefficient $\langle \alpha \rangle$ determined in the corresponding nonconjugate problem of convective heat transfer (i.e., without account for the thermal effect of the wall).

In [2–4], as a continuation of the general analysis of [1], a simple approximate method is developed for calculating the value of ε for a wall that is a flat plate of thickness δ. The present work is devoted to the derivation of a computational relation for ε that would correlate three different geometric forms of the wall: a plate, a cylinder, and a sphere with internal heat sources.

The fluctuational component of the "true" heat transfer coefficient is prescribed in the form of a step function that depends only on the time:

$$1 + \psi = 1 + b_1 , \quad 0 \leq \tau/\tau_0 \leq 2\pi s , \quad 1 + \psi = 1 - b_2 , \quad 2\pi s \leq \tau/\tau_0 \leq 2\pi . \tag{6}$$

Here b_1 and b_2 are the amplitudes of the fluctuations ψ for the "active" (increased heat transfer rate) and "passive" (decreased heat transfer rate) periods; τ is the time; τ_0 is the full period of fluctuations; s is the asymmetry parameter. The condition of periodicity of ψ gives the following normalizing relation:

$$b_1 s = b_2 (1 - s) . \tag{7}$$

Thus, the case of purely time fluctuations (homogeneous over the entire heat transfer surface) of the heat transfer intensity with two free parameters is considered: asymmetry s and amplitude b_1 (or b_2).

Mathematical Description of the Problem. A solution of the heat conduction equation

$$c\rho \frac{\partial \vartheta}{\partial \tau} = \lambda \left(\frac{\partial^2 \vartheta}{\partial x^2} + \frac{p}{x} \frac{\partial \vartheta}{\partial x} \right) + q_V \tag{8}$$

can be represented, due to its linearity, in the form of a superposition of a stationary part $\langle \vartheta \rangle(x)$, which satisfies the equation

$$\lambda \left(\frac{d^2 \langle \vartheta \rangle}{dx^2} + \frac{p}{x} \frac{d \langle \vartheta \rangle}{dx} \right) + q_V = 0 \tag{9}$$

and the boundary condition $d \langle \vartheta \rangle / dx = 0$ at $x = 0$, and a fluctuational part $\hat{\vartheta}(x, \tau)$ described by the equation

$$c\rho \frac{\partial \hat{\vartheta}}{\partial \tau} = \lambda \left(\frac{\partial^2 \hat{\vartheta}}{\partial x^2} + \frac{p}{x} \frac{\partial \hat{\vartheta}}{\partial x} \right) . \tag{10}$$

Here τ is the time; x is the transverse coordinate; c, ρ, λ are the heat capacity, density, and thermal conductivity of the wall, respectively; $q_V = \text{const}$ is the volumetric density of heat sources; p is a geometric factor ($p = 0$ for a plate, $p = 1$ for a cylinder, and $p = 2$ for a sphere).

The stationary solution has the form

$$\langle \vartheta \rangle = \langle \vartheta_\delta \rangle + \frac{q_V [1 - (x/\delta)^2]}{2(1+p)\lambda}.$$

The periodic solution of Eq. (10) satisfying the boundary condition $(\partial\hat{\vartheta}/\partial x)_{x=0} = 0$ has the following form:

a) $p = 0$ (a plate)

$$\hat{\vartheta} = \sum_{k=1}^{\infty} B_k \frac{\operatorname{sh}\sqrt{k/2}\,(1+i)\,\bar{x}}{\operatorname{sh}\sqrt{k/2}\,(1+i)\,\bar{\delta}}\, e^{ikt} + B_k^* \frac{\operatorname{sh}\sqrt{k/2}\,(1-i)\,\bar{x}}{\operatorname{sh}\sqrt{k/2}\,(1-i)\,\bar{\delta}}\, e^{-ikt}; \tag{11}$$

b) $p = 1$ (a cylinder)

$$\hat{\vartheta} = \sum_{k=1}^{\infty} B_k \frac{\operatorname{ber}_0\sqrt{k}\,\bar{x} + i\operatorname{bei}_0\sqrt{k}\,\bar{x}}{\operatorname{ber}_0\sqrt{k}\,\bar{\delta} + i\operatorname{bei}_0\sqrt{k}\,\bar{\delta}}\, e^{ikt} + B_k^* \frac{\operatorname{ber}_0\sqrt{k}\,\bar{x} - i\operatorname{bei}_0\sqrt{k}\,\bar{x}}{\operatorname{ber}_0\sqrt{k}\,\bar{\delta} + i\operatorname{bei}_0\sqrt{k}\,\bar{\delta}}\, e^{-ikt}; \tag{12}$$

c) $p = 2$ (a sphere)

$$\hat{\vartheta} = \frac{\delta}{x} \sum_{k=1}^{\infty} B_k \frac{\operatorname{sh}\sqrt{k/2}\,(1+i)\,\bar{x}}{\operatorname{sh}\sqrt{k/2}\,(1+i)\,\bar{\delta}}\, e^{ikt} + B_k^* \frac{\operatorname{sh}\sqrt{k/2}\,(1-i)\,\bar{x}}{\operatorname{sh}\sqrt{k/2}\,(1-i)\,\bar{\delta}}\, e^{-ikt}. \tag{13}$$

Here $t = \tau/\tau_0$, $\bar{x} = x/\sqrt{a\tau_0}$ are the dimensionless values of the time and the transverse coordinate; a is the thermal diffusivity of the wall; $\delta = L/(p+1)$; $L = V/F$ is a characteristic length; V, F is the volume and heat transfer surface of the heat- transmitting body, respectively; B_k, B_k^* are complex conjugate eigenvalues.

Omitting intermediate calculations similar to those given in [3, 4], we will write down the final relation for the relative quantity $\varepsilon = \alpha_m/\langle \alpha \rangle$:

$$\varepsilon = \varepsilon_{\min} + (1 - \varepsilon_{\min})\,\varepsilon_* . \tag{14}$$

Here

$$\varepsilon_{\min} = \frac{(1 - b_2)\,[b_2 + (1 - b_2)\,s]}{b_2 + (1 - 2b_2)\,s} \tag{15}$$

is the minimum value of ε;

$$\varepsilon_* = [s(2 - s) + (1 - s)^2 A \operatorname{cth} A]^{-1}, \tag{16}$$

is the value of ε calculated for the limiting case of an adiabatic "passive" period $b_2 = 1$, $\varepsilon_{\min} = 0$;

$$A = \frac{1}{2}\,\frac{\langle \alpha \rangle \tau_0}{\rho c \delta_*}\,\operatorname{cth}\left(\frac{\delta}{\delta_*}\right) \tag{17}$$

is the parameter of the thermal effect of the wall,

$$\delta_*^2 = \frac{s(1 - s)}{2}\, a\tau_0 \tag{18}$$

is the effective thickness of the wall.

Relations (14)-(18) determine the thermal effect of the wall on the averaged heat transfer and have the following asymptotics:

a) degeneration of the thermal effect of the wall ($\varepsilon \to 1$, $\alpha_m \to \langle \alpha \rangle$) occurs in the limiting cases $s \to 0$, $s \to 1$, $A \to 0$ (either the "active" or the "passive" period tends to zero; the thermal conductivity of the wall tends to infinity);

b) the maximum effect of the wall on the heat transfer ($\varepsilon_* \to 0$, $\varepsilon \to \varepsilon_{\min}$, $\alpha_m \to \varepsilon_{\min}\langle \alpha \rangle$) corresponds to $A \to \infty$ (zero thermal conductivity of the wall);

c) for $\delta/\delta_* \gtrsim 1$ ("semi-infinite" body) we obtain the case of degeneration of the effect of the thickness on the heat transfer (cth $(\delta/\delta_*) \to 1$);

d) when $\delta/\delta_* \to 0$ (an infinitely thin wall), the effect of the thermal conductivity of the wall degenerates ($A \to (1/2)(\langle \alpha \rangle \tau_0/\rho c\delta)$).

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